Prospect theory implications and limitations for the internationalization performance link

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Abstract

We advance a formal representation of cumulative prospect theory that disentangles two interdependent, yet different, components of subjectively perceived changes in wealth: reference points and objective outcomes. Our main result characterizes the conditions for what we call asymmetric comparative risk attitudes. That is, decision makers above reference point display stronger attitudes towards risk than decision makers below reference point. As an immediate consequence, decision makers below reference point prefer higher expected outcome prospects than decision makers above reference point. Our findings contradict the underlying assumption shared by most prospect theory applications in strategic management literature that allude to prospect theory's supposedly general prediction of risk aversion above and risk seeking below reference point. Numerical results from a multi agent computational model are provided to illustrate the behavioral consequence of asymmetric comparative risk attitudes.

1 Introduction

It has been argued that the question of whether there is a systematic relationship between the internationalization of firms and their performance is central to the field of international business (Glaum and Oesterle, 2007: 308). Explicitly or implicitly, the question is supposed to be a major element of all contributions to the theory of foreign direct investment and to other theories of foreign market entry. Although the study of the internationalization performance link has occupied a prominent place in the international management literature, the empirical findings are overall inconsistent (Sullivan, 1994; Ruigrok and Wagner, 2003). Only recently, Hennart (2007) and Verbeke et al. (2009) have pointed to weaknesses in the theoretical and methodological framework suggesting that there is no reason to expect a systematic relationship between internationalization and subsequent performance.

In this paper, we critically examine the theoretical rationale to support the opposite argument that there is a systematic relationship between performance and subsequent internationalization. E.g., Wennberg and Holmquist (2008) provide empirical evidence for a negative relationship between performance and subsequent internationalization suggesting that decision makers in low performing firms are more willing to accept the heightened risk associated with foreign direct investments. This line of argument derives from the organizational risk taking literature and has been guided primarily by two theories: the behavioral theory of the firm (Cyert and March, 1963) and prospect theory (Kahneman and Tversky, 1979). Although both theories are fundamentally different, research in the field of organizational risk taking has emphasized the similarities between these two theories, noting that both theories predict risk aversion when firm performance is above a reference point and risk propensity when firm performance is below a reference point for performance (e.g., Audia

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and Greve, 2006: 84). In this paper, however, we show that prospect theory does actually not support this line of argument: Risk attitudes cannot be infered from performance relative to reference points but remain subject to objective outcome distribution. We formally derive a set of prospect theory limitations and implications to provide a foundation for more thorough applications of prospect theory in strategic management literature in general and internationalization performance literature in particular.

Prospect theory and a behavioral theory of the firm assume that decision makers focus attention on a reference point for performance and that the outcome of a choice is determined as much by its contrast with a reference point as by the intrinsic value of the outcome itself. In prospect theory, this reference point generally corresponds to the status quo, whereas in the behavioral theory of the firm, the reference point is determined by social or historical comparison. Early studies in organizational risk taking literature typically measure risk with income stream uncertainty and calculate mean and variance of return on assets or equity over some time period for each firm within an industry. Using industry mean performance as reference point, firms within an industry are divided into those with performance below and those with performance above reference point. The groups are then be used to determine whether negative or positive risk return relations predominate. Studies usually report a negative and positive association between risk and return for firms below and above reference point and allude to a behavioral theory of the firm (Singh, 1986; Bromiley, 1991; Miller and Leiblein, 1996; Greve, 1998; Palmer and Wiseman, 1999; Deephouse and Wiseman, 2000) or prospect theory's supposedly general prediction of risk aversion above and risk seeking below a reference point (Bowman, 1980, 1982, 1984; Fiegenbaum and Thomas, 1988; Yegmin and Thomas, 1989; Fiegenbaum, 1990; Miller and Bromiley, 1990; Jegers, 1991; Sinha, 1994; Wiseman and Bromiley, 1996; Wiseman and Catanach, 1997; Don, 1997; Lehner, 2000; Fiegenbaum and Thomas, 2004).[1]

Our paper, in contradistinction, shows that prospect theory does not present a theoretical rationale to support the proposition driving previous studies since it neither implies that decision makers above reference point are necessarily risk averse nor that the degree of risk attitude increases with distance from the reference point. This result is noteworthy since more recent attempts to apply aforementioned prospect theory line of argument to the field of strategic management, e.g., resource expansion (Audia and Greve, 2006), mergers and acquisitions (Shimizu, 2007), and internationalization (Wennberg and Holmquist, 2008), indicate that it is still appropriate to infer individual risk attitudes from performance relative to a reference point. Our main result describes the precise conditions for what we call asymmetric comparative risk attitudes in prospect theory. That is, decision makers above reference point display stronger attitudes towards risk than decision makers below reference point. The new condition reveals an unexplored effect in prospect theory: Decision makers below reference point systematically prefer higher expected outcome prospects than decision makers above reference point.

Although Kahneman and Tversky (1979) offered a mathematical model, prospect theory applications in organizational risk taking literature have taken a qualitative approach to the theory. We hope that this paper will help to clarify some of the earlier misunderstandings and to reconcile prospect theory with strategic management literature in general and internationalization performance literature in particular. The remainder of this paper is organized as follows. A formal representation of prospect theory and related definitions are set out in section two. Section three derives the main results for cumulative prospect theory in general and Tversky and Kahneman (1992) power utility in particular. Section four applies a parameterized version of power utility to a multi agent computational model. The proof of our main result appears in the appendix.

[1]See Nickel and Rodriguez (2002) for a comprehensive review on organizational risk taking literature

2 Formal representation of cumulative prospect theory and nonzero attainment discrepancy

2.1 Reference dependent preferences

This section reviews the formal representation of cumulative prospect theory. The first part introduces prospect theory in general and the concept of nonzero attainment discrepancy in particular. The second part reviews some preliminary notation with a particular focus on loss aversion and diminishing sensitivity as distinct properties of Kahneman and Tversky (1979, 1992) utility functions.

Prospect theory (Kahneman and Tversky, 1979) has emerged as one of the most prominent alternatives to von Neumann and Morgenstern's (1944) expected utility. It combines empirically important properties of human risk taking that offer significant predictive improvement over expected utility, e.g., subjective probability weighting (Edwards, 1954; Handa, 1977), reference dependent preferences (Markowitz, 1952), and loss aversion (Kahneman and Tversky, 1979). The most widely applied variant of prospect theory is cumulative prospect theory (Starmer and Sugden, 1989; Tversky and Kahneman, 1992; Wakker and Tversky, 1993) that belongs to the larger family of rank dependent utility (Quiggin, 1982).

Cumulative prospect theory is even more general than rank dependent utility by allowing for sign dependence and reference dependence. The latter implies that under cumulative prospect theory the outcome of a choice is determined as much by its contrast with a reference point as by the intrinsic value of the outcome itself. That is, prospect theory decision makers interpret objective outcomes as gains and losses relative to some individual reference point. If the reference point corresponds to a decision maker's current wealth, the carriers of utility coincide with the objective outcomes that are received or paid. If the reference point, however, corresponds to an expectation level that differs from the status quo, while the difference is referred to as attainment discrepancy, the carriers of utility are subjectively perceived gains and losses rather than objectively reported changes in wealth. In this paper, we disentangle these interdependent, yet distinct components of carriers of utility to account for the separate roles of objective outcomes and attainment discrepancy in individual decision making.

Kahneman and Tversky (1979) have suggested that several factors, such as status quo, social norms, and aspiration levels may determine the reference point. However, it is still unclear how the reference point is formed and updated, given each of these factors. This paper does not address the question of what constitutes the reference point but illustrates the behavioral consequences of nonzero attainment discrepancy given an arbitrarily determined reference point. Most prospect theory applications in theoretical research assume for convenience that the reference point corresponds to status quo and, thus make no distinction between objective and subjectively perceived outcomes (Wakker and Zank, 2002; Schmidt, 2003; Köbberling and Wakker, 2005; Kyle et al., 2006; Baucells and Heukamp, 2006; Abdellaoui et al., 2007; Schmidt and Zank, 2008; Schmidt et al., 2008; Rieger and Wang, 2008; Bleichrodt et al., 2009, Schmidt and Zank, 2009).[2] A formal representation of attainment discrepancy effects in cumulative prospect theory is, to our knowlegde, missing up to now. However, there is theoretical and empirical evidence that reference points systematically depart from status quo. E.g., Lant (1992) supposes that reference points generally adapt to

[2] Wakker and Zank (2002) propose a simple preference foundation based on a weakening of comonotonic independence and constant proportional risk aversion of cumulative prospect theory with power utility, Schmidt (2003) advances an axiomatization of cumulative prospect theory which allows for variable reference points, Köbberling and Wakker (2005) propose a decomposition of risk aversion into three distinct components basic utility, probability weighting, and loss aversion, Kyle et al. (2006) analyze a liquidation problem and derive the reference point from the break even point, Baucells and Heukamp (2007) extend the second order stochastic dominance condition to cumulative prospect theory, Abdellaoui et al. (2007) propose a method to quantify the degree of loss aversion, Schmidt et al. (2008) propose third generation prospect theory, Schmidt and Zank (2008) characterize the precise conditions for risk aversion in cumulative prospect theory, Rieger and Wang (2008) extend cumulative prospect theory to continous probability distributions, Bleichrodt et al. (2009) give preference foundations for a segregated approach to additive utility in prospect theory, Schmidt and Zank (2009) propose an axiomatization of linear cumulative prospect theory

performance at a slower rate of change than change in performance and, thus implicitly expects zero attainment discrepancy to be rather the exception than the rule. Köszegi and Rabin (2006) plausibly suggest that a person's reference point is his or her recent expectations held about future outcomes rather than current assets. More recent empirical evidence suggests that individuals adjust reference points upwards as performance rises, but tend not to adjust their reference points downwards when performance declines which provides a natural account for negative attainment discrepancy (Arkes et. al, 2008).

Kahneman and Tversky (1979: 274) argue that the reference point will typically be the current asset position, but they allow the possibility that "the location of the reference point, and the consequent coding of outcomes as gains or losses, can be affected by the formulation of the offered prospects, and by the expectations of the decision maker". E.g., imagine a person with current assets ω and reference point $\omega^\dagger$ where $\omega, \omega^\dagger \in \mathbb{R}$. Then $z = \omega - \omega^\dagger$ denotes attainment discrepancy as the difference between current wealth and reference point for wealth.[3] For $z = 0$, the reference point corresponds to current assets in which case gains and losses coincide with the actual amounts that are received or paid. For $z \neq 0$, objective outcomes are coded relative to a reference point that differs from the status quo. In this case, prospect theory linearly transforms objective outcomes $\{x_1, \ldots, x_n\}$ into subjectively perceived gains and losses $\{x_1+z, \ldots, x_n+z\}$. Kahneman and Tversky (1979: 286-287) refer to $z > 0$ and $z < 0$ as positive and "negative translation of a choice problem, such as arises from incomplete adaptation to recent [gains or] losses". Throughout, we shall say that $z > 0$ and $z < 0$ denote decision makers above and below their reference points, respectively. More generally, nonzero attainment discrepancy $z \in \mathbb{R}$ represents a common increment in objective outcomes. Schmidt and Zank (2009) show that under linear cumulative prospect theory, e.g., $v''(x) = 0$ for $x \in \mathbb{R}$, preference rankings are independent of common increments. Note that Kahneman and Tversky (1979, 1992) utility as defined in this paper excludes independence of common increments.

2.2 Formal representation of cumulative prospect theory

It is generally assumed that a prospect consists of mutually exclusive outcomes, given a finite set of states of nature. Furthermore, it is assumed that exactly one state obtains, which is unknown to the decision maker. Let I be a finite set of states of nature; subsets of I are called events. Let X be a set of consequences, also called outcomes, identified with the set of real numbers $\mathbb{R}$. Let Y be an associated set of prospects. An uncertain prospect $y \in Y$ is a function from I into X that assigns to each state $i \in I$ a consequence $y(i) = x \in X$ with probability p. A prospect y can be written as a pair of n dimensional vectors $\{\mathbf{x}; \mathbf{p}\}$ with $x_1, \ldots, x_n \in X$ and nonnegative probabilities p_i that sum to one. Within this notation we implicitly assume that outcomes are ranked in increasing order and let $x_1 \leq x_k < 0 \leq x_{k+1} \leq \ldots \leq x_n$ for some $k \in \{0, \ldots, n\}$. Note that the prospect involves only negative or only positive outcomes if $k = n$ or $k = 0$, respectively. It is further assumed that individuals have preferences over prospects and we use the conventional notation $\succ$, $\succeq$, and $\sim$ to represent the relations of strict preference, weak preference, and indifference. A preference function $V(x, p)$ is a representing function or representation for $\succeq$ if it maps prospects to the reals such that $y_1 \succeq y_2 \Leftrightarrow V(y_1) \geq V(y_2)$. If a representing function exists then $\succeq$ is a weak order. Cumulative prospect theory holds if a representation exists of the form

$$V(x,p) = \sum_{i=1}^{n} \pi(p_i) v(x_i) \tag{1}$$

where $\pi(p)$ denotes decision weight of probability p and $v(x)$ denotes utility of outcome x. As already mentioned, the formal representation in this paper considers objective outcomes

[3] Lewin et al. (1944) define attainment discrepancy as the difference between prior aspiration level and actual performance achieved; Levinthal and March (1981) and Lant (1992) similarly calculate attainment discrepancy by subtracting aspiration level from actual performance

with additive attainment discrepancy as the relevant carriers of utility. Cumulative prospect theory defined in eq. (1) is written as $V(x,p) = \sum_{i=1}^{n} \pi(p_i)v(x_i+z)$ where $z \in \mathbb{R}$ throughout. It is easily verified that the choice situation involves only losses or only gains if $z \leq -x_n$ or $z \geq -x_1$, respectively. More generally, we shall say that the choice situation involves nonmixed outcomes if $x_i + z \leq 0$ or $x_i + z \geq 0$ for $i = 1, \ldots, n$ with a strict inequality for at least one i. Our notational convention implies further that $z \notin]-x_n, -x_1[$ is a necessary and sufficient condition for nonmixed outcome prospects. Otherwise $z \in]-x_n, -x_1[$ implies mixed outcome prospects. It is generally assumed that real life prospects involve some possibility of gain and some possibility of loss (MacCrimmon and Wehrung, 1990; March and Shapira, 1987). Therefore, we restrict attainment discrepancy to $z \in]-x_n, -x_1[$ unless otherwise noted.

The probability transformation function $\pi(p)$ maps the interval of probabilities $[0,1]$ into itself to convert objective probabilities p into subjective decision weights $\pi(p)$ as first proposed by Edwards (1962) and extended by Handa (1977). Since $\pi(p)$ is generally not linear, prospect theory may violate first order stochastic dominance (Fishburn, 1978), an assumption that many theorists are reluctant to give up (e.g., Machina, 1982: 292; Quiggin, 1982: 327). This disadvantage has motivated the development of cumulative prospect theory and variants on it (Starmer and Sugden, 1989; Tversky and Kahneman, 1992; Wakker and Tversky, 1993). Throughout, we refer to cumulative prospect theory in the sense of Tversky and Kahneman (1992) based on Chateauneuf and Wakker's (1999) preference axiomatization for decisions under risk. Cumulative prospect theory combines the descriptive realism of first generation prospect theory with the transformation of cumulative probabilities rather than single probabilities. The transformation of cumulative probabilities was introduced in rank dependent utility theory (Quiggin, 1982) and is consistent with first order stochastic dominance. Cumulative prospect theory is even more general than rank dependent utility by allowing for sign dependence and reference dependence. That is, there exist separate probability weighting functions for losses and gains, and utility is defined on gains and losses rather than on final states. Decision weight of probability p_i associated with a particular state of nature $i \in I$ is defined as

$$\pi(p_i) = \begin{cases} \pi^+(\sum_{j=i}^{n} p_j) - \pi^+(\sum_{j=i+1}^{n} p_j), & i > k \\ \pi^-(\sum_{j=1}^{i} p_j) - \pi^-(\sum_{j=1}^{i-1} p_j), & i \leq k \end{cases} \tag{2}$$

where $\pi^+(p)$ and $\pi^-(p)$ increase in p with $\pi^+(0) = \pi^-(0) = 0$ and $\pi^+(1) = \pi^-(1) = 1$.[4]

In what follows, we review the properties of the utility function applied throughout this paper. Note that the utility function in cumulative prospect theory generally agrees with the utility function in first generation prospect theory. Formally, we define $\wp_p$, the family of Kahneman and Tversky (1979, 1992) utility functions.

Definition 1 (Kahneman and Tversky (1979, 1992) utility)**.** *Let $v(x) \in \wp_p$ be a Kahneman and Tversky (1979, 1992) utility function. Then for all $x \in \mathbb{R}$, $v(x)$ is*

1.1 strictly convex for $x < 0$ with $v'(x) > 0$ and $v''(x) > 0$

1.2 strictly concave for $x > 0$ with $v'(x) > 0$ and $v''(x) < 0$

1.3 steeper for losses than for gains with $v'(-x) > v'(x)$ where $x > 0$

Properties (1.1) and (1.2) relate to s-shaped Kahneman and Tversky (1979, 1992) utility functions.[5] As an immediate consequence, it follows that $\lim_{x\to\infty \wedge x\to-\infty} v^{(n)}(x) = 0$ where $v^{(n)}(x)$ denotes the n^{th} order derivate of $v(x)$ at x with $n \in \mathbb{N}$, provided $v^{(n)}(x)$ exists.

[4] We follow the common convention that $\sum_{j=i}^{i-1} p_j = 0$

[5] Note that Kahneman and Tversky (1979: 278-279) assume strict convexity $>$ and strict concavity $<$ in their seminal publication while they suppose convexity $\geq$ and concavity $\leq$ in the formal representation of cumulative prospect theory (Tversky and Kahneman, 1992: 303)

The latter reflects the principle of diminishing sensitivity for losses and gains, "that is, the marginal value of both gains and losses generally decreases with their magnitude" (Kahneman and Tversky, 1979: 278). More specifically, "the difference in value between a gain of 100 and a gain of 200 appears to be greater than the difference between a gain of 1,100 and a gain of 1,200. Similiarly, the difference between a loss of 100 and a loss of 200 appears greater than the difference between a loss of 1,100 and a loss of 1,200". Note that diminishing sensitivity is entirely defined in terms of utility and does not involve probability weighting.

Definition 2 (Diminishing sensitivity)**.** *Suppose that cumulative prospect theory holds. Let $v(x) \in \wp_p$ be a Tversky and Kahneman (1979, 1992) utility function. Assume further that either $x_i - s \geq 0$ or $x_{i+1} + s \leq 0$ where $s \geq 0$. Then $v(x_i) - v(x_i - s) + v(x_{i+1}) - v(x_{i+1} + s) > 0$ or $v(x_i) - v(x_i - s) + v(x_{i+1}) - v(x_{i+1} + s) < 0$, respectively.*

Property (1.3) is a direct consequence of loss aversion as defined in the original version of prospect theory (Kahneman and Tversky, 1979: 279). That is, the disutility from losses looms larger than the utility from equal sized gains. Moreover, the aversiveness of symmetric fair bets generally increases with the size of the stake. This clearly is a behavioral concept entirely defined in terms of preferences and not necessarily linked to prospect theory in general or cumulative prospect theory in particular.

Definition 3 (Loss aversion)**.** *Define $y_1 = \{(x_1, x_2, \ldots, x_k, x_{k+1}, \ldots, x_{n-1}, x_n); (\mathbf{p})\}$ and $y_2 = \{(x_1 + s, x_2, \ldots, x_k, x_{k+1}, \ldots, x_{n-1}, x_n - s); (\mathbf{p})\}$ where $x_1 = -x_n$, $s > 0$ and $p_i = p_n$ for $i = 1, \ldots, n$. Then loss aversion holds if $y_2 \succ y_1$.*

Schmidt and Zank (2005) have shown that under cumulative prospect theory loss aversion is not only determined by the curvature of the utility function but also by the curvature of the probability weighting function. Suppose that cumulative prospect theory holds. Substitution of eq. (1) and elimination of common terms implies that loss aversion holds if and only if $\frac{\pi(p_1)}{\pi(p_n)} > \frac{v(x_n) - v(x_n - s)}{v(x_1 + s) - v(x_1)}$ where $x_1 + s = -(x_n - s)$ and $x_n - s > 0$. It turns out that under cumulative prospect theory utility being steeper for losses than for gains is neither necessary nor sufficient for loss aversion. Rather it remains subject to the degree of probabilistic loss aversion defined analogously to Wakker's (1994) concept of probabilistic risk aversion. Then the decision weight of a loss always exceeds the decision weight of an equally probable gain (Schmidt and Zank, 2008; Zank, 2008). Here $\pi(p_1) > \pi(p_n)$. For infinitisemal $s \to 0$ it follows that loss aversion reduces to $\frac{\pi(p_1)}{\pi(p_n)} \frac{v'(x_1)}{v'(x_n)} > 1 \Leftrightarrow \frac{\pi(p_1)}{\pi(p_n)} v'(x_1) - v'(x_n) > 0$ where $x_1 = -x_n$.

The next section derives the main results of this paper for the family of Kahneman and Tversky (1979, 1992) utility functions in general and for a parameterized version of cumulative prospect theory proposed by Tversky and Kahneman (1992) in particular.

3 Attainment discrepancy effects in cumulative prospect theory

3.1 Risk aversion

The first part of this section derives a sufficient condition for risk aversion with respect to nonzero attainment discrepancy $z \in \mathbb{R}$. As a result, decision makers are risk averse if loss aversion holds and if attainment discrepancy weakly exceeds the midrange of objective outcomes defined as the arithmetic mean of the maximum and minimum value in $\{x_1, \ldots, n_n\}$. This result is noteworthy for at least two reasons: First, it contradicts the underlying assumption shared by most prospect theory applications in strategic management literature that allude to prospect theory's supposedly general prediction of risk aversion above and risk seeking below a reference point. Second, it is of particular importance for the main result of this paper derived in the next part of this section. That is, decision makers with positive attainment discrepancy display stronger attitudes towards risk than decision makers with equal sized negative attainment discrepancy, referred to as asymmetric comparative risk attitude. For simplicity of exposition, we restrict the analysis to equally probable events and let $p_i = p_n$ for $i = 1, \ldots, n$. Note that this paper does not aim at a necessary and sufficient condition of risk aversion. Rather it is based on an analysis of the interrelationship between attainment discrepancy and objective outcome distribution to derive a sufficient condition for risk aversion in the first place. Hereon, this condition is used to characterize the precise condition for asymmetric comparative risk attitudes in cumulative prospect theory. While the first and the second part derive the results for the family of Kahneman and Tversky (1979, 1992) utility functions, the third part derives the results for the parameterized version of cumulative prospect theory proposed by Tversky and Kahneman (1992).

First, let us clarify the behavioral property of risk aversion that we adopt in this paper. One way to define risk aversion is in the weak sense (Pratt, 1964: 124). Weak sense risk aversion holds if the expected outcome of a prospect for certain is always prefered to the prospect itself. In this paper, risk aversion is defined in the strong sense. Rothschild and Stiglitz (1970) define strong risk aversion as aversion to mean preserving spreads. E.g., risk averse decision makers disprefer a prospect with positive variance and expected outcome over an alternative prospect with identical expected outcome but smaller variance. For any n dimensional mixed outcome prospect $y_1 = \{(x_1, \ldots, x_n); (\mathbf{p})\}$ where $k \in \{1, \ldots, n-1\}$ such that $x_k < 0 < x_{k+1}$, it follows that individuals are risk averse if for all $\delta x > 0$ it follows that

$$\left\{ \left(x_1 + \frac{\delta x}{p_1}, x_2, \ldots, x_k, x_{k+1}, \ldots, x_{n-1}, x_n - \frac{\delta x}{p_n} \right); \Big(\mathbf{p} \Big) \right\} \succeq y_1 \tag{3}$$

whenever $p_1, p_n > 0$. For $p_1 = p_n$ it follows further that $s = \frac{\delta x}{p_1} = \frac{\delta x}{p_n}$ represents a mean preserving spread in the sense of Rothschild and Stiglitz (1970: 227-231).[6] Recall that due to our notation s must be chosen such that rank ordering of outcomes is maintained and $x_1 + s < x_2$ and $x_n - s > x_{n-1}$. Define $y_2 = \{(x_1 + s, x_2, \ldots, x_k, x_{k+1}, \ldots, x_{n-1}, x_n - s); (\mathbf{p})\}$. Then risk aversion holds if $y_2 \succ y_1 \Leftrightarrow V(y_2) > V(y_1)$. Suppose that cumulative prospect theory holds. Substitution of eq. (1) and elimination of common terms implies that

$$RA(\mathbf{x}, \mathbf{p}, z, s) = \frac{\pi(p_1)}{\pi(p_n)} \left[v\Big(x_1 + z + s \Big) - v\Big(x_1 + z \Big) \right] + v\Big(x_n + z - s \Big) - v\Big(x_n + z \Big) \tag{4}$$

denotes risk aversion (RA). Individuals are risk seeking if $RA(\mathbf{x}, \mathbf{p}, z, s) < 0$, risk averse if

[6] Schmidt and Zank (2008) note that mean preserving spreads involve splitting of consequences and, thus empirical tests of strong risk aversion may be problematic if coalescing is violated. Birnbaum (2004, 2005) and Birnbaum and Navarette (1998) indicate that splitting good consequences tends to increase the attractiveness of a lottery, while splitting bad consequences tends to decrease the attractiveness. Hence, in empirical studies it may be difficult to disentangle these effects from the effects of strong risk aversion.

$RA(\mathbf{x},\mathbf{p},z,s) > 0$, and risk neutral if $RA(\mathbf{x},\mathbf{p},z,s) = 0$ while $|RA(\mathbf{x},\mathbf{p},z,s)|$ is referred to as degree of risk attitude. Note that eq. (2) implies that $\pi(p_1) \gtreqless \pi(p_n)$ even though $p_1 = p_n$.

For $z \notin]-x_n, -x_1[$ it follows from Schmidt and Zank (2008) that decision makers are necessarily risk averse if and only if the probability weighting function is convex in the domain of gains and concave in the domain of losses and if the utility function is concave in the domain of gains and also in the domain of losses. If the choice situation, however, involves gains and losses any necessary and sufficient condition of risk aversion remains subject to the gain loss exchange rate implied by loss aversion. In what follows, we drop all restrictions on the shape of the probability weighting function but impose loss aversion to derive a sufficient condition for risk aversion. Suppose that $z \in [-(x_n - s), -(x_1 + s)]$ such that $x_1 + z + s \leq 0 \leq x_n + z - s$ and let $x_1 + z \geq -(x_n + z) \Leftrightarrow x_1 + z = -(x_n + z) + \Delta$ where $\Delta \geq 0$. Without loss of generality, we assume infinitisemal $s \to 0$. Since Kahneman and Tversky (1979, 1992) utility functions are continuous and twice differentiable, it follows that left and right side derivatives are identical and $v'_{\downarrow}(x) = v'_{\uparrow}(x)$ whenever $x \neq 0$. Taking limits when s approaches zero implies that eq. (4) reduces to

$$RA(\mathbf{x},\mathbf{p},z,s,\Delta) = s\left[\frac{\pi(p_1)}{\pi(p_n)}\frac{\delta v\big(-(x_n+z)+\Delta+s\big)}{\delta s} - \frac{\delta v(x_n+z-s)}{\delta s}\right] \tag{5}$$

where $z \in]-x_n, -x_1[$. For $\Delta = 0$ it follows that $RA(\mathbf{x},\mathbf{p},z,s,\Delta) > 0$ if loss aversion holds. It follows further that $RA(\mathbf{x},\mathbf{p},z,s,\Delta)$ increases in Δ. Formally,

$$\frac{\delta RA(\mathbf{x},\mathbf{p},z,s,\Delta)}{\delta \Delta} = s\frac{\pi(p_1)}{\pi(p_n)}\frac{\delta^2 v\big(-(x_n+z)+\Delta+s\big)}{\delta s \delta \Delta} > 0. \tag{6}$$

The latter is an immediate consequence of the curvature of Kahneman and Tversky (1979, 1992) utility functions where $v''(-x) > 0$ for $x > 0$. Therefore, we shall say that risk aversion holds for all $z \in]-x_n, -x_1[$ such that $z \geq -\big(\frac{x_1+x_n}{2}\big)$ if loss aversion holds.

Corollary 1 (Risk aversion)**.** *Suppose that cumulative prospect theory holds with $v(x) \in \wp_p$. Then risk aversion holds for all $z \in]-x_n, -x_1[$ such that $z \geq -\big(\frac{x_1+x_n}{2}\big)$ if loss aversion holds.*

Note that the above corollary is a sufficient but not necessary condition for risk aversion. Decision makers are necessarily risk averse for all $z \geq -\frac{x_1+x_n}{2}$. However, the reversed implication does not hold in general. That is, decision makers are not necessarily risk seeking for all $z < -\frac{x_1+x_n}{2}$. More generally, we shall say that under prospect theory risk aversion cannot be infered from attainment discrepancy for at least two reasons. First, decision makers with negative or positive attainment discrepancy are not necessarily risk seeking or risk averse, respectively. Second, the degree of risk attitude cannot be infered from attainment discrepancy. That is, decision makers with higher attainment discrepancy are not necessarily more risk averse than decision makers with lower attainment discrepancy. E.g., for $z \in]-x_1, -x_n[$ it follows from eq. (5) that $RA(\mathbf{x},\mathbf{p},z,s,\Delta)$ increases in attainment discrepancy. Formally,

$$\frac{\delta RA(\mathbf{x},\mathbf{p},z,s,\Delta)}{\delta z} = s\left[\frac{\pi(p_1)}{\pi(p_n)}\frac{\delta^2 v\big(-(x_n+z)+\Delta+s\big)}{\delta s \delta z} - \frac{\delta^2 v(x_n+z-s)}{\delta s \delta z}\right] > 0 \tag{7}$$

where $-(x_n + z) + \Delta + s \leq 0 \leq x_n + z - s$. The latter is again a direct consequence of the general properties of Kahneman and Tversky (1979, 1992) utility functions where $v''(-x) > 0$ and $v''(x) < 0$ for $x > 0$. If the choice situation, however, involves non-mixed outcome prospects, the degree of risk attitude decreases in attainment discrepancy and approaches zero as absolute attainment discrepancy becomes infinitely large. Formally, $\lim_{z\to\infty \wedge z\to -\infty} |RA(\mathbf{x},\mathbf{p},z,s,\Delta)| = 0$ whenever $z \notin]-x_1, -x_n[$ while the latter is an immediate consequence of diminishing sensitivity. Then $RA(\mathbf{x},\mathbf{p},z,s,\Delta)$ is not a linear function in $z \in \mathbb{R}$.

Fig. 1 displays $RA(\mathbf{x}, \mathbf{p}, z, s, \Delta)$ defined in eq. (5) as a function in attainment discrepancy. It can be easily verified that decision makers with negative attainment discrepancy are not necessarily risk seeking and that decision makers with higher absolute attainment discpreancy do not necessarily display stronger attitudes towards risk than decision makers with lower absolute attainment discrepancy. Note also that $z = -x_1$ and $z = -x_n$ are vertical asymptotes of $RA(\mathbf{x}, \mathbf{p}, z, s, \Delta)$ since $\lim_{z \to -x_1 \wedge z \to -x_n} RA(\mathbf{x}, \mathbf{p}, z, s, \Delta) = \pm\infty$. The latter is an immediate consequence of the fact that $v(x) \in \wp_p$ is not continuously differentiable at $x = 0$ where $v'(0) = \infty$.

Corollary 2 (Degree of risk attitude)**.** *Suppose that cumulative prospect theory holds with $v(x) \in \wp_p$. Then the degree of risk attitude $|RA(\mathbf{x}, \mathbf{p}, z, s)|$ increases or decreases in absolute attainment discrepancy $|z|$ if $z \in]-x_1, -x_n[$ or $z \notin]-x_1, -x_n[$, respectively.*

Our findings so far contradict the underlying assumption shared by most prospect theory applications in strategic management literature that allude to prospect theory's supposedly general prediction of risk aversion above and risk seeking below a reference point while risk is supposed to increase with distance from the reference point (Bowman, 1980, 1982, 1984; Fiegenbaum and Thomas, 1988; Yegmin and Thomas, 1989; Fiegenbaum, 1990; Miller and Bromiley, 1990; Jegers, 1991; Sinha, 1994; Wiseman and Bromiley, 1996; Wiseman and Catanach, 1997; Don, 1997; Lehner, 2000; Fiegenbaum and Thomas, 2004). These studies typically measure risk with income stream uncertainty and calculate mean and variance of return on assets or equity over some time period for each firm within an industry. Using industry mean performance as reference point, firms within an industry are divided into those with performance below and those with performance above reference point. The groups are then be used to determine whether negative or positive risk return relations predominate. Studies usually report a negative and positive association between risk and return for firms below and above reference point and conclude that firms below and above reference point are risk seeking and risk averse, respectively. However, prospect theory obviously does not present a theoretical rationale to support the proposition driving these studies since it neither implies that decision makers above reference point are necessarily risk averse nor that the degree of risk attitude increases with distance from the reference point.

Some authors have extended the prospect theory argument and suggested that risk contributes negatively and positively to subsequent performance if decision makers perceive themselve below and above reference point, respectively (Bowman, 1984; Miller and Bromiley, 1990; Wiseman and Bromiley, 1996; Wiseman and Catanach, 1997). This argument goes as follows: Risk averse decision makers are only willing to accept an increase in income stream risk if an investment opportunity offers high expected returns, which implies that risk contributes positively to subsequent performance. Risk seeking, in turn, is associated with having given up returns to obtain increased risk, which implies that for decision makers below reference point, increases in risk will decrease subsequent performance. As a result, this line of research draws on prospect theory to predict diverging performance trends for firms below and above reference point. The next part of this section shows that prospect theory actually implies the opposite, converging performance trends for decision makers below and above a common reference point. The latter is an immediate consequence of asymmetric comparative risk attitudes derived below.

3.2 Asymmetric comparative risk attitudes

We now state and proof the main result of this paper. That is, decision makers with positive attainment discrepancy display stronger attitudes towards risk than decision makers with equal sized negative attainment discrepancy if the midrange of objective outcomes is weakly positive. Let $CRA(\mathbf{x}, \mathbf{p}, z, s) = |RA(\mathbf{x}, \mathbf{p}, z, s)| - |RA(\mathbf{x}, \mathbf{p}, -z, s)|$ where $z > 0$ denote comparative risk attitudes (CRA). Suppose that loss aversion holds. Let $x_1 \geq -x_n \Leftrightarrow x_1 = -x_n + \Delta$ where $\Delta \geq 0$. Corollary 1 implies that $RA(\mathbf{x}, \mathbf{p}, z, s)$ is necessarily positive while $RA(\mathbf{x}, \mathbf{p}, -z, s)$ can be positive or negative. Then $CRA(\mathbf{x}, \mathbf{p}, z, s) > 0$ if and only if

$RA(\mathbf{x}, \mathbf{p}, z, s) > RA(\mathbf{x}, \mathbf{p}, -z, s)$ and $RA(\mathbf{x}, \mathbf{p}, z, s) > -RA(\mathbf{x}, \mathbf{p}, -z, s)$. Taking limits when s approaches zero implies that $CRA(\mathbf{x}, \mathbf{p}, z, s) > 0$ if and only if

$$\frac{\pi(p_1)}{\pi(p_n)}\left[v'\big(-x_n + \Delta + z\big) - v'\big(-x_n + \Delta - z\big)\right] > v'(x_n + z) - v'(x_n - z) \tag{8}$$

and

$$\frac{\pi(p_1)}{\pi(p_n)}v'(-x_n + \Delta + z) - v'(x_n - z) > -\left[\frac{\pi(p_1)}{\pi(p_n)}v'(-x_n + \Delta - z) - v'(x_n + z)\right] \tag{9}$$

where $v'(x) = \frac{\delta v(\mathbf{x},z,s)}{\delta s}$ and $-x_n + \Delta + z < 0 < x_n - z$. The former inequality (8) is an immediate consequence of s-shaped Kahneman and Tversky (1979, 1992) utility where $v''(-x) > 0$ and $v''(x) < 0$ for $x > 0$. The latter inequality (9) holds for all $\Delta \geq 0$ if loss aversion holds. Again, fig. 1 shows that $|RA(\mathbf{x}, \mathbf{p}, z, s)| > |RA(\mathbf{x}, \mathbf{p}, -z, s)|$, $z > 0$.

Corollary 3 (Asymmetric comparative risk attitudes)**.** *Suppose that cumulative prospect theory holds with $v(x) \in \wp_p$. Then $CRA(\mathbf{x}, \mathbf{p}, z, s) > 0$ for all $z \in]0, Min\{-x_1, x_n\}[$ if loss aversion holds and if $x_1 \geq -x_n$.*

Next, we derive the behavioral implications of the corollary above. That is, decision makers with negative attainment discrepancy make systematically more promising decisions than decision makers with equal sized positive attainment discrepancy. Let us associate the most promising decision with the prospect that maximizes expected outcome identified by a preference function of the form $E(\mathbf{x}, \mathbf{p}) = p\sum_{i=1}^{n} x_i$. More generally, we shall say that any preference function of the form $E(\mathbf{x}, \mathbf{p}, z) = p\sum_{i=1}^{n}(x_i + z)$ where $z \in \mathbb{R}$ maximizes expected outcome since linear transformation of objective outcomes is isotone and, therefore order preserving. Expected outcome maximizers are obviously indifferent between both prospects y_1 and y_2 because $E(y_2) - E(y_1) = 0$ for all $z \in \mathbb{R}$. As an immediate consequence, decision makers with positive attainment discrepancy depart more severe from the ranking order that maximizes expected outcome than decision makers with equal sized negative attainment discrepancy whenever $CRA(\mathbf{x}, \mathbf{p}, z, s) > 0$. The remainder of this section elaborates on the behavioral implications of asymmetric comparative risk attitudes in more detail and advances the central proposition of this paper.

Recall that both prospects y_1 and y_2 offer the same expected outcome. Now suppose that decision makers are either risk averse or risk seeking and, therefore weakly prefer $\succeq$ one prospect over the other. Assume further that either the prefered prospect depreciates or that the disprefered prospect appreciates by an arbitrarily small amount ε. Then reversing the preference order obviously increases expected outcome in either case. In what follows we show that decision makers with positive attainment discrepancy are more reluctant to preference reversal than decision makers with equal sized negative attainment discrepancy and, therefore systematically settle for lower expected outcome prospects. Suppose that loss aversion holds and that midrange of objective outcomes is weakly positive. Then corollary 1 implies that decision makers with positive attainment discrepancy are necessarily risk averse while decision makers with negative attainment discrepancy can be risk averse or risk seeking.

First, suppose that both decision makers are risk averse and, therefore prefer y_2 over y_1. Define two prospects $\varphi^- = \{(-\varepsilon, \ldots, -\varepsilon); (\mathbf{p})\}$ and $\varphi^+ = \{(\varepsilon, \ldots, \varepsilon); (\mathbf{p})\}$ where $\varepsilon > 0$. Then $y_2 \oplus \varphi^-$ offers a smaller expected outcome than y_1 where the symbol $\oplus$ means that objective outcomes are added while keeping the probability distribution fixed. Correspondingly, $y_1 \oplus \varepsilon^+$ offers a higher expected outcome than y_2. For all $\varepsilon \geq s$ it follows further that $F_{y_2 \oplus \varphi^-}(x) \geq F_{y_1}(x)$ for all $x \in \mathbb{R}$ where F_y denotes the cumulative distribution function of prospect y. Therefore, we shall say that prospect y_1 first order stochastically dominates prospect $y_2 \oplus \varphi^-$ for all $\varepsilon \geq s$. Because cumulative prospect theory satisfies first order stochastic dominance we expect decision makers to prefer y_1 over $y_2 \oplus \varphi^-$ and, thus maximize expected outcome for all $z \in \mathbb{R}$. For $\varepsilon < s$, however, we find that decision makers with positive attainment discrepancy reveal a stronger preference for $y_2 \oplus \varphi^-$ over y_1 and also for y_2 over $y_1 \oplus \varphi^+$

than decision makers with equal sized negative attainment discrepancy. Let $V(y_2 \oplus \varphi^-|z)$ denote the preference rank of prospect $y_2 \oplus \varphi^-$ given attainment discrepancy z where $z > 0$. Then $V(y_2 \oplus \varphi^-|z) - V(y_1|z) > V(y_2 \oplus \varphi^-|-z) - V(y_1|-z)$ and $V(y_2|z) - V(y_1 \oplus \varphi^+|z) > V(y_2|-z) - V(y_1 \oplus \varphi^+|-z)$. Substitution of eq. (1) implies that

$$\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s+z-\varepsilon) - v(x_1+z) + v(x_1-z) - v(x_1+s-z-\varepsilon)\Big] \\ +\Big[v(x_n-s+z-\varepsilon) - v(x_n+z) + v(x_n-z) - v(x_n-s-z-\varepsilon)\Big] > 0 \qquad (10)$$

and

$$\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s+z) - v(x_1+z+\varepsilon) + v(x_1-z+\varepsilon) - v(x_1+s-z)\Big] \\ +\Big[v(x_n-s+z) - v(x_n+z+\varepsilon) + v(x_n-z+\varepsilon) - v(x_n-s-z)\Big] > 0 \qquad (11)$$

where $s > \varepsilon$. Note that the latter inequalities (10) and (11) are an immediate consequence of diminishing sensitivity.

Now, suppose that decision makers with negative attainment discrepancy are risk seeking and, thus prefer prospect y_1 over y_2. If the prefered prospect depreciates, we find that decision makers with negative attainment discrepancy are rather indifferent between y_2 and $y_1 \oplus \varphi^-$ while decision makers with positive attainment discrepancy display relatively more preference for $y_2 \oplus \varphi^-$ than y_1. If the disprefered prospect appreciates, decision makers with negative attainment discrepancy are again rather indifferent between $y_2 \oplus \varphi^+$ and y_1 while decision makers with positive attainment discrepancy display relatively more preference for y_2 than for $y_1 \oplus \varphi^+$. Formally, $V(y_2 \oplus \varphi^-|z) - V(y_1|z) > V(y_1 \oplus \varphi^-|-z) - V(y_2|-z)$ and $V(y_2|z) - V(y_1 \oplus \varphi^+|z) > V(y_1|-z) - V(y_2 \oplus \varphi^+|-z)$. Then

$$\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s+z-\varepsilon) - v(x_1+z)\Big] + v(x_n-s-z) - v(x_n-z-\varepsilon) > \\ -\left\{\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s-z) - v(x_1-z-\varepsilon)\Big] + v(x_n-s+z-\varepsilon) - v(x_n+z)\right\} \qquad (12)$$

and

$$\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s+z) - v(x_1+z+\varepsilon)\Big] + v(x_n-s-z+\varepsilon) - v(x_n-z) > \\ -\left\{\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s-z+\varepsilon) - v(x_1-z)\Big] + v(x_n-s+z) - v(x_n+z+\varepsilon)\right\} \qquad (13)$$

where $s > \varepsilon$. As shown in the appendix, the latter inequalities (12) and (13) are an immediate consequence of loss aversion and s-shaped curvature of Kahneman and Tversky (1979, 1992) utility.

Proposition 1. *Suppose that cumulative prospect theory holds with $v(x) \in \wp_p$. Then decision makers with negative attainment discrepancy prefer higher expected outcome prospects than decision makers with equal sized positive attainment discrepancy if loss aversion holds and if midrange of objective outcomes is weakly positive.*

The interest in a particular parametric form for utility has led to corresponding derivations of cumulative prospect theory. The next section derives corollaries 1 through 3 from a parameterized version of cumulative prospect theory proposed by Tversky and Kahneman (1992).

3.3 Parametric utility

In what follows, we derive corollaries 1 through 3 for Tversky and Kahneman's (1992) power utility. Our formal representation is based on a new condition recently advanced by al-Nowaihi et al. (2008). As a result, inequality (19) derived below presents a sufficient condition for risk aversion with respect to loss aversion, attainment discrepancy, and objective outcome distribution. Note that inequality (19) is independent of any particular value s. We conclude this section with a brief discussion on the validity of aforementioned corollaries 1 through 3 for cumulative prospect theory with power utility. Hereon, section four applies Tversky and Kahneman's (1992) power utility to a multi agent computational model to illustrate the behavioral implications of our central proposition in more detail.

Researchers have specified parameterized versions of prospect theory and the parameters have been estimated from experimental data; see Neilson and Stowe (2002) for a review on different parameterizations of prospect theory. Throughout, we refer to the parameterized version of cumulative prospect theory proposed by Tversky and Kahneman (1992) which is the most used parametric form in empirical and theoretical applications (many reference are given in Wakker and Zank, 2002). Then

$$\pi(p_i) = \frac{P_i^{\varphi}}{(P_i^{\varphi} + (1-P_i)^{\varphi})^{\frac{1}{\varphi}}} - \frac{(P_i - p_i)^{\varphi}}{((P_i - p_i)^{\varphi} + (1-(P_i - p_i))^{\varphi})^{\frac{1}{\varphi}}} \tag{14}$$

denotes decision weight for probability p_i where $\varphi = \delta$ if $x_i + z \geq 0$ and $\varphi = \gamma$ if $x_i + z < 0$. Note that $\pi(p)$ transforms cumulative rather than single probabilities and $P_i = \sum_{j=i}^{n} p_j$ if $x_i + z \geq 0$ and $P_i = \sum_{j=1}^{i} p_j$ if $x_i + z < 0$ for $i = 1, \ldots, n$. Assuming $\gamma = \delta = 1$ leads to a linear probability weighting function where $\pi(p) = p$.[7] Utility is defined as a two part power function of the form

$$v(x+z) = \begin{cases} (x+z)^{\alpha}, & x+z \geq 0 \\ -\lambda\big(-(x+z)\big)^{\beta}, & x+z < 0. \end{cases} \tag{15}$$

where $\lambda > 0$ and $0 < \alpha, \beta < 1$. Let $v^{(n)}(x+z)$ denote the n^{th} order derivative at $x+z$, provided $v^{(n)}(x+z)$ exists. Formally,

$$v^{(n)}(x+z) = \begin{cases} (x+z)^{\alpha-n} \prod_{i=0}^{n-1} (\alpha - i), & x+z \geq 0 \\ \frac{\lambda\left(-(x+z)\right)^{\beta-n}}{(-1)^{n-1}} \prod_{i=0}^{n-1} (\beta - i), & x+z < 0 \end{cases} \tag{16}$$

where $n \in \mathbb{N}$. Then Taylor series expansion of $v(x_i^{\dagger} + s)$ with $x_i^{\dagger} = x_i + z$ implies that risk aversion as defined in eq. (4) holds if and only if

$$\sum_{n=1}^{\infty} \left[\frac{\pi(p_1)}{\pi(p_n)} \frac{s^n}{n!} \frac{(-x_1^{\dagger})^{\beta-n}\lambda}{(-1)^{n-1}} \prod_{i=0}^{n-1} (\beta - i) + \frac{(-s)^n}{n!} (x_n^{\dagger})^{\alpha-n} \prod_{i=0}^{n-1} (\alpha - i) \right] > 0. \tag{17}$$

With $(-s)^n = -\frac{s^n}{(-1)^{(n-1)}}$ it follows further that risk aversion holds if

$$\frac{\pi(p_1)}{\pi(p_n)} \frac{s^n}{n!} \frac{\lambda(-x_1^{\dagger})^{\beta-n}}{(-1)^{n-1}} \prod_{i=0}^{n-1} (\beta - i) - \frac{s^n}{n!} \frac{(x_n^{\dagger})^{\alpha-n}}{(-1)^{n-1}} \prod_{i=0}^{n-1} (\alpha - i) > 0 \tag{18}$$

for all $n \in \{1, \ldots, \infty\}$. Note that eq. (18) presents a sufficient but not necessary condition for risk aversion. That is, decision makers are necessarily risk averse if inequality (18) holds

[7] For $0 < \gamma, \delta < 1$ it follows that $\pi^{+}(p)$ and $\pi^{-}(p)$ are inverse s-shaped and put higher weights on lower probabilities and lower weights on higher probabilities as orignally proposed by Kahneman and Tversky (1979)

for all $n = 1, \ldots, \infty$. However, the reversed implication does not hold insofar as risk aversion does not imply that inequality (18) holds for all $n = 1, \ldots, \infty$.

Only recently, al-Nowaihi et al. (2008) have shown that utility being steeper for losses than for gains requires that $\lambda > 1$ and $\alpha = \beta$. For Kahneman and Tversky (1979, 1992) utility it follows that eq. (18) reduces to

$$\Lambda - \big(\kappa \cdot g\big)^{\alpha - n} > 0 \tag{19}$$

for $n \in \{1, \ldots, \infty\}$ where $\Lambda = \lambda \cdot \frac{\pi(p_1)}{\pi(p_n)}$, $\kappa = \big[\frac{x_n + z}{x_n} \cdot \frac{-x_1}{-(x_1+z)}\big]$, and $g = \frac{x_n}{-x_1}$. Note that Λ denotes loss aversion while g represents midrange of objective outcomes insofar as $\frac{x_1 + x_n}{2} \geq 0 \Leftrightarrow g \geq 1$ and $\frac{x_1+x_n}{2} < 0 \Leftrightarrow g < 1$. For all $z \in]-x_n, -x_1[$ such that $x_1 + z < 0 < x_n + z$ it follows that κ represents attainment discrepancy insofar as $z \geq 0 \Leftrightarrow \kappa \geq 1$ and $z < 0 \Leftrightarrow \kappa < 1$. It is easily verified that corollaries 1 through 3 apply to Tversky and Kahneman's (1992) power utility. Assume that loss aversion holds such that $\Lambda > 1$. Then risk aversion holds for all $z \in]-x_n, -x_1[$ such that $x_1 + z \geq -(x_n + z)$. Also, risk aversion increases in attainment discrepancy as implied by corollary 2. Finally, asymmetric comparative risk attitudes hold for $\kappa_1 < 1 < \kappa_2$ and $g \geq 1$ such that $\Lambda - \big(\kappa_2 \cdot g\big)^{\alpha-n} > \Lambda - \big(\kappa_1 \cdot g\big)^{\alpha-n}$ and $\Lambda - \big(\kappa_2 \cdot g\big)^{\alpha-n} > \big(\kappa_1 \cdot g\big)^{\alpha-n} - \Lambda$. Verification is left to the reader.

The next section applies Tversky and Kahnemann's (1992) power utility to a multi agent computational model to illustrate the behavioral implications of our central proposition in more detail. As shown below, asymmetric comparative risk attitudes in prospect theory imply oscillating performance trends among multiple decision makers below and above a common reference point.

4 Numerical results from multi agent computation

This section presents results from a multi agent computational model. First, we introduce the general model set up and the particular parameter values used in the simulation. Then we discuss the results provided in table 1 and figure 2.

The simulation models the choice behavior of prospect theory decision makers, also referred to as agents. The sequence of events is as follows: A finite set of prospects is made available to an agent while both the outcome vector and the probability vector of all prospects are known. The reference point is computed and compared with current wealth to determine attainment discrepancy. Then a decision is made on what prospect to choose. Finally, the decision maker receives the outcome associated with the state of nature that obtains.

Suppose there are $a = 1, \ldots, m$ prospect theory decision makers with identical preferences best described by Tversky and Kahneman's (1992) parameterized version of cumulative prospect theory. There is a sequence of rounds $t = 1, \ldots, T$ where T is finite. Let $w_{a,t}$ and $x_{a,t}$ denote wealth and outcome of decision maker a at time t where $w_{a,t} = w_{a,t-1} + x_{a,t-1}$. In particular, we assume that decision makers use mean wealth $\overline{w_t} = \sum_{a=1}^{m} \frac{w_{a,t}}{m}$ as reference point at time t. The choice situation in each round $t \in \{1, \ldots, T\}$ involves a finite set of n dimensional mixed outcome prospects where each event $i = 1, \ldots, n$ resolves with probability p_i such that $\sum_{i=1}^{n} p_i = 1$. Suppose that objective outcomes $\widetilde{x}_i \sim U(x_1, x_n)$ are drawn from a continuous uniform distribution where $x_1 < 0 < x_n$ and assume further that industry dynamics are temporarily stable and best described by the first two moments of the outcome distribution. Note that any particular prospect can be selected by more than one decision maker. For $m = 2$ decision makers, it follows that both decision makers have equal absolute attainment discrepancy $z_{1,t} = -z_{2,t} = |z_{a,t}|$ at any particular time $t \in \{1, \ldots, T\}$. Since both decision makers apply the same preference function to an identical set of prospects, the model allows us to measure the isolated effect of asymmetric risk attitudes on overall risk preferences and decision outcome. Note also that the model set up is consistent with the general assumptions in prospect theory applications in strategic management literature insofar as it describes the choice behavior of identical decision makers below and above a common reference point.

We run the model several times while each run has a length of $T = 30.000$ rounds. The choice situation in each round involves two mixed outcome prospects and six events, three of which are associated with strictly negative outcomes while the remaining three events offer strictly positive outcomes. We choose six dimensional prospects to minimize computational run time on the one hand and decrease the likelihood of stochastically dominated prospects on the other. Negative and positive outcomes are drawn from $U(x_1-|z|, -|z|)$ and $U(|z|, x_n+|z|)$, respectively, to make sure that each choice situation involves the same number of subjectively perceived gains and losses. We set $x_1 = -(1-c)\cdot 200$ and $x_n = (1+c)\cdot 200$ while c is varied by 0.25 from 0.0 to 0.5. Note that the midrange of objective outcome distribution is zero for $c = 0$, positive for $c > 0$, and negative for $c < 0$. Initial wealth is set to $w_{1,1} = 500$ and $w_{2,1} = 550$ so that $\overline{w_1} = 525$. Events resolve with equal probability $p_i = p_{cons.}$ for $i = 1, \ldots, n$ while $n \cdot p_{cons.} = 1$.

In what follows, we briefly discuss the parameters used for the probability weighting (14) and utility function (15). Following Barberis et al. (2001) and, more recently, Bromiley (2009), our model applies the parameters from Tversky and Kahneman's (1992) experimental results. Tversky and Kahneman (1992) found median values of 0.88 for both α and β. The latter is obviously in line with the condition recently advanced by al-Nowaihi (2008). Therefore, model estimates keep $\alpha = \beta = 0.88$. Tversky and Kahneman (1992) estimated λ for each of their subjects and found λ had a 25^{th} percentile of 4.74, median of 2.25, and 75^{th} percentile of 1.1 so our model uses values of 1.1, 2.25, and 4.74 for λ. By using the 25^{th}, 50^{th}, and 75^{th} percentile values, the model corresponds to normal values that are appropriate for the center 50% of the population. Note that Abdellaoui et al. (2007) estimated quite similar parameters for prospect theory's value function. They estimated the parameters separately for each subject and found median estimates of 0.725 for α and 0.717 for β. Likewise, they found mean estimates of λ, the loss aversion parameter, between 2.04 and 2.64 across subjects, and median estimates from 1.69 to 1.97.

For the probability weighting functions, Tversky and Kahneman (1992) found for positive and negative weighting parameters $\delta = 0.61$ and $\gamma = 0.69$, respectively. Unfortunately, there is no parameterization of δ and γ that is consistent with probabilistic loss aversion if the choice situation involves more dimensional prospects. E.g., probabilistic loss aversion holds for prospects with $n = 3$ events, one of which is associated with a fixed outcome of zero, whenever $\delta > \gamma$ (Zank, 2008).[8] In this paper, we let $\delta = \gamma = 1$ and, thus suppose that probabilities are not distorted to ensure that loss aversion holds with $\Lambda = \frac{\pi(p_1)}{\pi(p_n)}\lambda > 1$ for all rounds. Previous researchers have argued that in cases of a symmetric probability distribution, where each of the outcomes has an equal probability, probabilities are not distorted. E.g., Quiggin (1982), who was the first one to propose that cumulative probabilities are distorted rather than the raw individual probabilities, suggests that at $p = \frac{1}{2}$ there will be no distortion of probabilities. He explains this partly because it has the intuitively appealing property that symmetric probabilities are undistorted by probability weighting. According to this logic, Levy and Levy (2002) argue that the probability of any symmetrical bet (with a $\frac{1}{n}$ probability for each of the n outcomes) should not be distorted. This is also a result of Viscusi's (1989) prospective reference theory with a symmetric reference point, for which he finds experimental support. However, not all authors agree with the fact that symmetric probability distributions are undistorted (see Luce, 2000: 84-100).

Next, we discuss the results provided in table 1 and fig. 2. Recall that the choice situation in each round involves not more than two prospects with six events while the model does not control for stochastically dominated prospects. As an immediate consequence, decision makers with identically parameterized preference functions will prefer the same prospect for most of the rounds. E.g., results from the first run where $c = 0.5$ and $\lambda = 1.10$ show that

[8] Zank (2008) shows that $\delta > \gamma$ implies that $\pi(p_1) > \pi(p_n)$ for small and moderate probabilities p_1, p_n. This statement, however, may not be valid for all probabilities

		attainment discrepancy								
		$z > 0$			$z < 0$			$P(T <= t)$ two tail		
		Loss aversion (λ)			Loss aversion (λ)			Loss aversion (λ)		
p	**x**	1.10	2.25	4.74	1.10	2.25	4.74	1.10	2.25	4.74
$p_{cons.}$	$c = 0.50$	61.56	49.46	25.22	64.83	68.70	51.75	0.81	0.10	0.01
	$c = 0.25$	25.22	20.71	18.13	30.06	33.46	46.12	0.76	0.28	0.03
	$c = 0.00$	-12.21	-12.61	-3.95	-11.50	11.26	11.95	0.98	0.53	0.42

Table 1: Mean outcome for decision makers above and below reference point

both decision makers prefer the same prospect in 29,221 of 30,000 rounds. Table 1 reports the mean outcome for the remaining 779 rounds where both decision makers prefered different prospects. For the first run, it follows from table 1 that a decision maker with negative attainment discrepancy yields a mean outcome of 64.83 while an identical decision maker with equal sized positive attainment discrepancy yields a mean outcome of not more than 61.56. However, we cannot reject the hypotheses that the means of the two groups are equal for a significance level smaller than $\alpha < 0.1$. The third column in table 1 reports the corresponding p-value from a two-sample t-test for unequal variances. As shown in table 1, the mean outcome is generally higher for decision makers with negative attainment discrepancy than for decision makers with equal sized positive attainment discrepancy. Despite the fact that sample means are not significantly different in some samples, table 1 supports our basic proposition: Decision makers with negative attainment discprepancy prefer prospects with higher expected outcome than identical decision makers with equal sized positive attainment discrepancy while the effect increases in midrange of objective outcome distribution, c, and degree of loss aversion, λ. Note that significance levels increase as we increase the number of events per prospect or the number of prospects per round.

As a result, asymmetric comparative risk attitudes imply oscillating performance trends for decision makers below and above a common reference point. Fig. 2 displays the wealth differerence between both decision makers $\Delta = w_{2,t} - w_{1.t}$ at time t if $c = 0$ and $\lambda = 2.25$. Performance trends obviously do not diverge. Rather they temporarily converge and oscillate around a common trend.

5 Conclusion

We have introduced the concept of asymmetric comparative risk attitudes and its coherent behavioral implication. As a result, decision makers with positive attainment discrepancy display stronger attitudes towards risk than identical decision makers with equal sized negative attainment discrepancy if loss aversion holds and if the midrange of objective outcome distribution is weakly positive. The new condition leads to an unexplored effect in prospect theory: Decision makers below reference point systematically prefer higher expected outcome prospects than decision makers above.

Our findings are unrelated to a behavioral description and remain subject to careful empirical verification. Rather, they are tautologic insofar as they derive from a set of empirically validated properties of cumulative prospect theory. Therefore, our results are limited to the assumption that prospect theory is a valid theory of choice for mixed outcome prospects. However, there are aspects of behavior that are problematic for prospect theory in general and s-shaped utility in particular. E.g., as with all rank dependent theories, cumulative prospect theory rests on the principle of comonotonic independence (Schmeidler, 1989). While earlier studies indicated mixed evidence regarding the descriptive validity of this principle (Wakker et al., 1994), Birnbaum (2004, 2005) and Birnbaum and Navarette (1998) have identified a number of behavioral patterns that refute the rank dependent formulation. See also the review of Marley and Luce (2005) who concluded that these violations rule out cumulative prospect theory as a descriptive model of behavior. Moreover, if the choice situation simultaneously involves gains and losses, the validity of prospect theory remains subject to gain loss separability. However, recent empirical research rejects the double matching axiom, a necessary condition for gain loss separability (Birnbaum and Bahra, 2006; Wu and Markle, 2008). Furthermore, the s-shaped utility function hypothesis is based on experiments in which subjects are asked to choose separately between alternatives with either only positive or only negative outcomes, while most of these studies employ a certainty equivalent approach. More recent experiments with mixed gambles provide at least conflicting empirical results whether or not s-shaped utility is a valid model of choice between pairs of mixed outcome prospects (Levy and Levy, 2002; Wakker, 2003; Baltussen et al., 2006). Moreover, the certainty equivalent approach remains problematic since it has been documented that a certain outcome has a dramatic effect on subjects' choices (Battalio et al., 1990), e.g., Schneider and Lopes (1986) find support for s-shaped preferences only when a prospect with a riskless component is involved.

Our findings contradict the underlying assumption shared by most prospect theory applications in organizational risk taking literature that allude to prospect theory's supposedly general prediction of risk averion above and risk seeking below a reference point while risk is supposed to increase with distance from the reference point. This result is noteworthy since more recent attempts to apply aforementioned prospect theory line of argument to the field of strategic management, e.g., resource expansion (Audia and Greve, 2006), mergers and acquisitions (Shimizu, 2007), and internationalization (Wennberg and Holmquist, 2008), indicate that it is still appropriate to infer individual risk attitudes from performance relative to a reference point. However, our results reveal that prospect theory in general and cumulative prospect theory in particular imply rather complex attitudes towards risk and do not present a theoretical rationale to support the proposition driving most of these studies. We hope that this paper will help to reconcile prospect theory with strategic management and related disciplines and to clarify some of the earlier misunderstandings.

In providing a foundation for future research, our findings suggest a number of other promising directions. The concept of asymmetric comparative risk attitudes reveals a paradox of success insofar as past success itself may set the stage for dysfunctional persistence. Here this result derives from individual attitudes towards risk determined by distinct properties of cumulative prospect theory. Our formal argument implies that coalescence of loss aversion with risk aversion leads decision makers to act overly risk averse while coalescence of loss aversion with risk propensity leads decision makers to act only moderately risk seeking. In the former case, risk averse decision makers dimiss almost any prospect with positive expected outcome while, in the latter, risk seeking decision makers occasionally accept prospects with negative expected outcome. As a result, decision makers with negative attainment discrepancy systematically prefer higher expected outcome prospects than decision makers with equal sized positive attainment discrepancy. The direct effect, however, remains subject to a more comprehensive descriptive underpinning. First and foremost, our finding calls for empirical research regarding the validity of asymmetric comparative risk attitudes and related second order effects in cumulative prospect theory. More combinatory research applying findings from behavioral research, e.g., goal setting theory (Locke and Latham, 2004), must be investigated to extend the corresponding behavioral line of argument. To the extent that real life prospects involve some possibility of gain and some possibility of loss (MacCrimmon and Wehrung, 1990; March and Shapira, 1987) and that prospect theory is a valid theory of choice for mixed outcome prospects, the new concept of asymmetric comparative risk attitudes should bring professional managers to consider reference dependency as a major fraction in observed decision outcome.

6 Appendix

Lemma 1 (Eq. (12)). *Suppose that cumulative prospect theory holds with $v(x) \in \wp_p$. Assume further that loss aversion holds and $x_1 \geq -x_n$. Then*

$$\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s+z-\varepsilon)-v(x_1+z)\Big]+v(x_n-s-z)-v(x_n-z-\varepsilon)>$$
$$-\left\{\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s-z)-v(x_1-z-\varepsilon)\Big]+v(x_n-s+z-\varepsilon)-v(x_n+z)\right\}.$$

where $s \geq \varepsilon \geq 0$.

Proof 1. *Let $x_1 = -x_n$. For $\varepsilon = 0$ it follows that inequality (12) holds if loss aversion holds. Note that the the left side of inequality (12) weakly increases in x_1 while the right side weakly decreases in x_1. Therefore, we shall say that inequality (12) holds for all $x_1 \geq -x_n$ if it holds for $x_1 = -x_n$. Note also that the left side of inequality (12) increases not more in ε than the right side. Formally, $-[c \cdot v'(-a+s) - v'(a)] < -[c \cdot v'(-b) - v'(b-s)]$ where $c = \frac{\pi(p_1)}{\pi(p_n)}$, $a = x_n - z$ and $b = x_n + z$ such that $0 < a < b$. Therefore, we shall say that inequality (12) holds for all $s \geq \varepsilon \geq 0$ if it holds for $s = \varepsilon$. Let $x_1 = -x_n$ and $s = \varepsilon$. Then*

$$c\Big[v(-b+s)-v(-b-s)\Big]+v(b-2s)-v(b)>0$$

which can be written as

$$c\big[v(-b+s)-v(-b)+v(-b)-v(-b-s)\big]+v(b-2s)-v(b-s)+v(b-s)-v(b)>0.$$

For infinitisemal $s \to 0$ it follows that

$$c\big[v'_{\downarrow}(-b)+v'_{\uparrow}(-b)\big]+v'_{\downarrow}(b-\delta x)-v'_{\uparrow}(b)>0.$$

Using the fact that left and right side derivatives are identical so that $v'_{\downarrow}(b) = v'_{\uparrow}(b)$ yields

$$0<2\cdot c\cdot v'(-b)+v'(b-\delta x)-v'(b). \quad \square$$

Lemma 2 (Eq. (13)). *Suppose that cumulative prospect theory holds with $v(x) \in \wp_p$. Assume further that loss aversion holds and $x_1 \geq -x_n$. Then*

$$\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s+z)-v(x_1+z+\varepsilon)\Big]+v(x_n-s-z+\varepsilon)-v(x_n-z)>$$
$$-\left\{\frac{\pi(p_1)}{\pi(p_n)}\Big[v(x_1+s-z+\varepsilon)-v(x_1-z)\Big]+v(x_n-s+z)-v(x_n+z+\varepsilon)\right\}$$

where $s \geq \varepsilon \geq 0$.

Proof 2. *Proof, which is analogous to proof 1, is omitted.*

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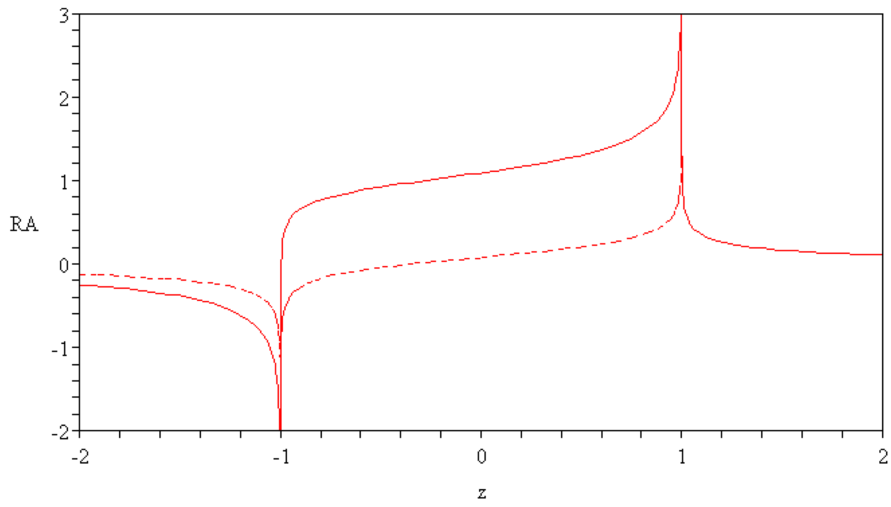

Figure 1: Risk aversion (RA) as defined in eq. (5) for $\frac{\pi(p_1)}{\pi(p_n)} = 1$, $\Delta = 0$ and $x_n = 1$ based on the parameterized version of cumulative prospect theory proposed by Tversky and Kahneman (1992) where $\alpha = \beta = 0.88$ and $\lambda = 2.25$ (solid) and $\lambda = 1.10$ (dotted)

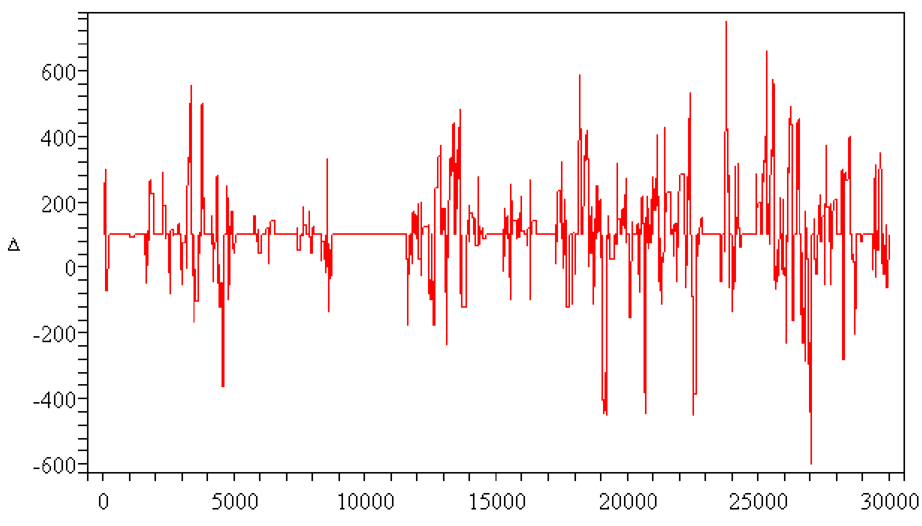

Figure 2: Wealth difference $\Delta_t = w_{2,t} - w_{1.t}$ between two identical prospect theory decision makers $i = 1, 2$ for $t \in \{1, \ldots, 30000\}$ rounds from computational model based on the parameterized version of cumulative prospect theory proposed by Tversky and Kahneman (1992) where $\alpha = \beta = 0.88$, $\lambda = 2.25$, and $\delta = \gamma = 1$