ANALYSIS AND FORECASTING OF THE DEVELOPMENT OF BANKING: THE ESTONIAN CASE

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Abstract

The main purpose of the paper is to test the possibilities of treating a bank as an enterprise that produces services and for which the same laws are valid (at least in Estonia) as for other enterprises. As Estonia is a small country, the banks here can be considered small or medium-sized, despite the high profitability of their enterprises.

Banks and other financial institutions compose a unique set of business firms whose assets and liabilities, regulatory restrictions, economic functions and operation make them an important subject of research. Banks' performance monitoring, analysis and control deserve special attention in respect to their operation and performance results from the viewpoint of various audiences such as investors/owners, regulators, customers, and management.

This paper presents two econometric models. In addition, whether the development of the Estonian banking agrees with R. Solow´s theory of balanced growth is considered.
Keywords: banking analysis, econometric models.

1. Theoretical background

One can ask what is the production or product of a bank? In our opinion, the product of the bank is the amount of the services, the volume of which can be measured by the total income of the bank, which is the measure of the amount of production (Aarma and Vainu 2003,2004,2005,2006).

We selected the total income of the banks (y) as the output variable (dependent variable) and used profit earning assets (x_1), equity (x_2), liabilities (x_3) and fixed assets (x_4) as factors (independent variables).

The time series were treated as consisting of three components:

(1) $$y(t) = f(t) + h(t) + e_t$$

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where $y(t)$ represents the actual time series;
$f(t)$ represents the linear trend in the time series;
$h(t)$ represents the harmonious component in the time series;
e_t represents residuals.

The harmonious component is determined by Fourier's series:

(2) $$h(t) = a_0 + \sum_{j=1}^{k} (a_j \cos\alpha + b_j \sin\alpha), \qquad \alpha = j\frac{t2\pi}{T}$$

where j represents the number of harmonious components,
t represents time,
T represents length of the time series (the number of periods).

We chose the power function as the type of the model.

(3) $$y = ax^{\alpha} z^{\beta}, \quad \alpha + \beta = 1.$$

To estimate the parameters a and α with the method of least squares, it was necessary to first find logarithms of the primary data. Then, according to the rules of analysing time series, we checked for the existence of a trend and harmonious component in the time series of the logarithms of the selected parameters.

We followed R. Solow's approach and assumed that the chosen factors can be regrouped so that two groups would be formed: profit earning current assets, $x = x_1 + x_2 + x_3$; and profit earning fixed assets, $z = x_4$.

(4) $$y = ax^{\alpha} z^{1-\alpha}.$$

Now we assume that part of the total income will be invested into profit earning current assets:

(5) $$I = sy = dx / dt$$

and that the fixed assets will remain unchanged for a certain period of time.

(6) $$z(t) = z_0, \; dz / dt = 0.$$

Now

(7) $$\frac{dx}{dt} = sy = sf(x,z) = sf(x,z_0).$$

Now let the ratio of current assets to fixed assets $k = x/z$; then

(8) $$x(t) = k(t)z_0.$$

Differentiating (8) on the basis of time, we obtain

(9) $$\frac{dz}{dt} = \frac{dk}{dt}z_0$$

and

(10) $$\frac{dk}{dt}z_0 = sf(x,z_0),$$

from which

(11) $$\frac{dk}{dt}z_0 = sz_0 f\left(\frac{x}{z_0},1\right)$$

and denoting $f\left(\frac{x}{z_0},1\right) = f(k)$,

we get

(12) $$\frac{dk}{dt} = sf(k).$$

Equation (12) shows that all investments are directed toward increasing the amount of profit earning current assets.
In the case of the power function

(13) $$\frac{dk}{dt} = sak^{\alpha}.$$

By integrating (13) we get

$$\int k^{-\alpha} dk = \int asdt$$

from which

(14) $$\frac{1}{1-\alpha}k^{1-\alpha} = ast + A.$$

To determine the constant A, we assume that $k(t) = k_0$, if $t = 0$.

$$A = \frac{1}{1-\alpha} k_0^{1-\alpha},$$

$$k^{1-\alpha} = ast(1-\alpha) + k_0^{1-\alpha},$$

(15) $$k(t) = \left[ast(1-\alpha) + sak^{1-\alpha}\right]\frac{1}{1-\alpha}.$$

The increment of the total income is found as follows:

(16)
$$\frac{dy}{dt} = \frac{d}{dt}\left[ax^{\alpha} z_0^{1-\alpha}\right] = \left[\alpha a x^{\alpha-1} z_0^{1-\alpha}\right]\frac{dx}{dt} = \alpha a \frac{x^{\alpha}}{x} z_0^{1-\alpha} \frac{dx}{dt} = \alpha y \frac{1}{x}\frac{dx}{dt},$$

(17) $$\frac{1}{y}\frac{dy}{dt} = \alpha \frac{1}{x}\frac{dx}{dt} = \alpha \frac{sy}{x} = \alpha sb,$$

where $b = y/x$ is the productivity of profit earning assets, the rate of increment of which is

(18)
$$\frac{1}{b}\frac{db}{dt} = \frac{x}{y}\frac{d}{dt}\left(\frac{y}{x}\right) = \frac{x}{y}\frac{1}{x}\frac{dy}{dt} - \frac{x}{y} y \frac{1}{x^2}\frac{dx}{dt} = \frac{1}{y}\frac{dy}{dt} - \frac{1}{x}\frac{dx}{dt} = (\alpha - 1)sb.$$

The rate of increment of the productivity of fixed assets is

(19) $$\frac{1}{v}\frac{dv}{dt} = \frac{1}{v}\frac{d}{dt}\left(\frac{y}{z_0}\right) = \frac{1}{v}\frac{1}{z_0}\frac{dy}{dt} = \frac{z_0}{y}\frac{1}{z_0}\frac{dy}{dt} = \frac{1}{y}\frac{dy}{dt} = \alpha sb.$$

Let us now examine the situation where the increase of fixed assets is linear:

(20) $$z(t) = a_0 + a_1 t.$$

Now the amount of the profit earning current assets is

(21) $$x(t) = k(t)z(t) = k(t)(a_0 + a_1 t)$$

and its increment is

(22) $$\frac{dx}{dt}=\frac{dk}{dt}(a_0+a_1t)+a_1k(t).$$

Assuming the existence of the function

(23) $$y=f(x,z)=f(x,a_0+a_1t),$$

we can write:

(24) $$\frac{dk}{dt}\left(a_0+a_1t\right)+a_1k_t=sf\left(x,a_0+a_1t\right),$$

from which

(25) $$\left(a_0+a_1t\right)\left(\frac{dk}{dt}+k_t\frac{a_1}{a_0+a_1t}\right)=s\left(a_0+a_1t\right)f\left(\frac{x}{a_0+a_1t},1\right)$$

or

(26) $$\frac{dk}{dt}=sf(k)-k\frac{a_1}{a_0+a_1t},$$

where $a_1/\left(a_0+a_1t\right)=n=\frac{1}{z}\frac{dz}{dt}$ is the increment rate of fixed assets.
The condition of equilibrium is here

(27) $$\frac{1}{x}\frac{dx}{dt}=\frac{1}{z}\frac{dz}{dt},$$

from which $m=s/n$, where m represents the ratio of current assets and total income.
As

(28) $$m=\frac{x}{y}=\frac{x}{zf(k)}=\frac{k}{f(k)},$$

then, in the case of equilibrium

(29) $$\frac{s}{n} = \frac{k}{f(k)}.$$

In the case of the Cobb-Douglas function

(30) $$\frac{dk}{dt} + nk = sak^{\alpha}.$$

Equation (30) is a first-order non-linear non-homogeneous differential equation the solution of which is the function

(31) $$k(t) = \left[\left(k_0^{1-\alpha} - \frac{as}{n}\right)e^{-n(1-\alpha)t} + \frac{as}{n}\right]^{\frac{1}{1-\alpha}}.$$

It can be seen from equation (31) that if $t \to \infty$, then $e^{-n(1-\alpha)t} \to 0$ and the ratio of current assets and fixed assets will move towards the equilibrium state $\left(\frac{as}{n}\right)^{\frac{1}{1-\alpha}}$.

2. Econometric models

Let us first construct two-factor power function (6 functions), of which the best was the function with the minimum standard error

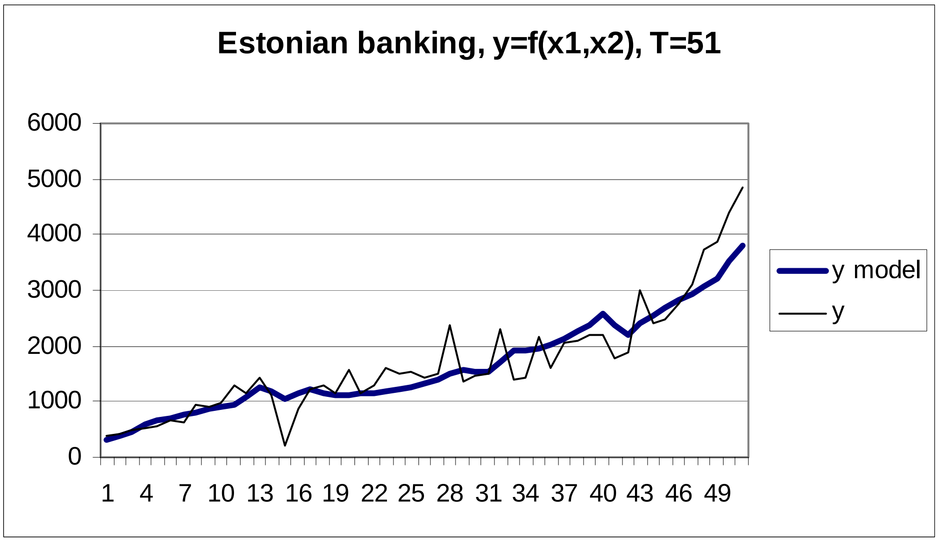
(32)
$$\begin{aligned}\ln y = &-0{,}1083 - 0{,}0221t + 0{,}2848\cos\alpha + 0{,}0076\sin\alpha + \\ &+ 0{,}1409\cos 2\alpha + 0{,}0983\sin 2\alpha - \\ &- 0{,}0689\cos 3\alpha - 0{,}0398\sin 3\alpha + 0{,}051\ln x_1 + 0{,}949\ln x_2\end{aligned}$$

(33)
$$\begin{aligned}y = &\,0{,}3301 x_1^{0{,}051} x_2^{0{,}949} \\ &\exp\begin{bmatrix}0{,}2848\cos\alpha + 0{,}0076\sin\alpha + 0{,}1409\cos 2a + \\ + 0{,}0983\sin 2\alpha - 0{,}0689\cos 3\alpha - 0{,}0398\sin 3\alpha - 0{,}0221t\end{bmatrix}\end{aligned}$$

R = 0,93.

The suitability of function (33) can be seen in Figure 1.

Figure 1. Suitability of the model $y = f(x_1, x_2)$.



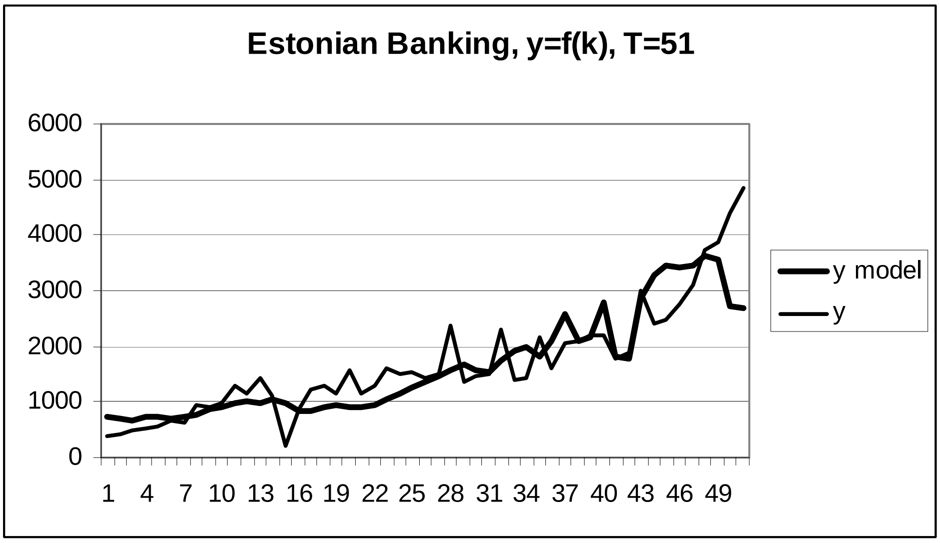
Source: Authors' calculations

Then we found the parameters of the function $y = f(k)$. As a result we obtained the function

(34) $$y = 79{,}41k^{0{,}81}, \quad R = 0{,}84.$$

The suitability of this model is demonstrated by Figure 2.

Figure 2. Suitability of the model $y = f(k)$.



Source: authors' calculations

Already the figures indicate that the forecasting capability of the models is low due to the rapid economic growth in Estonia in 2007.
Let us now examine the development of Estonian banking from the perspective of the theory of balanced growth. It can be seen from Equation (31) that the state of equilibrium can be calculated using the following formula:

$$y^{bal} = \left(\frac{as}{n}\right)^{\frac{1}{1-\alpha}}.$$

By using Equation (34) and taking $s = 0{,}1$, we get the value of the equilibrium state equal to 2,1845E+12 million Estonian kroon. Actually, the level of the Estonian banking was 4851,3 million Estonian kroon.
It is absolutely impossible to achieve such a level. The authors are of the opinion that. Solow's theory of balanced growth and "golden age" is just a beautiful concoction, whose materialization is as utopian as that of communism.

Conclusions

1. Econometric models can be used to analyse and prognosticate banking parameters; power functions give the best results.

2. Different functions give somewhat different results, but these differences are not large.
3. Analysis of the dynamics of Estonian banking from the perspective of the theory of balanced growth revealed that Estonian banking is far from a state of equilibrium.

Calculations show that balanced growth is a utopian theory.

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